MATHEMATICAL THEORIES AND PHILOSOPHICAL INSIGHTS IN COSMOLOGY

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Philosophers of science often regard physics as stemming from the interplay of theories and facts. By a *theory* we are to understand here the interpreted theory of a mathematical structure, *i.e.* a theory-*cum*-model; by a *fact*, a description of observations couched in such language that it can be seen to agree or to disagree with a theorem of the relevant theory. A classical statement of this view is contained in Max Born's lecture *Experiment and Theory in Physics*,[1] delivered in 1943. In it Born harshly criticized the British cosmologists, A. S. Eddington and E. A. Milne, for resorting to philosophical principles as basic premises in the construction of physical theories. In a letter to Born of September 7, 1944, Albert Einstein made the following comment on this lecture and on his friend's negative stance towards philosophical speculation in physics:

> *Ich habe mit viel Interesse Deinen Vortrag gegen die Hegelei gelesen, welche bei uns Theoretikern das Don Quijote'sche Element ausmacht oder soll ich sagen, den Verführer? Wo dies Übel oder Laster aber gründlich fehlt, ist der hoffnungslose Philister auf dem Plan.*[2]

Contrasting his own methodology of physics and philosophy of nature with that propounded by Born, Einstein added:

> *Ich [glaube] an volle Gesetzlichkeit in einer Welt von etwas objektiv Seiendem, das ich auf wild spekulativem Wege zu erhaschen suche.*[3]

Einstein's devotion to the 'Quixotic element' in science, his firm persuasion that facts alone cannot suffice as a source of guidance for the formulation and acceptance of theories is shown also in several other texts, one of which I shall have occasion to quote later.

General views concerning the nature of things, whether obtained through insight or by wild speculation, appear to be indispensable for choosing a physically viable theory from the luxuriant jungle of conceivable mathematical structures, infinitely many of which can be made to fit, within the allowable margin of imprecision, any given set of facts. Indeed, without some such views the physicist would be hard put to describe his observations in terms agreeable with a mathematical theory. The interplay between mathematical theories and what I propose to call philosophical insights - or, if you wish, philosophical hunches - is therefore no less essential for the growth of physics than the interplay between

theories and facts. Fortunately, philosophy of science has begun of late to direct to the former some of the attention it had paid to the latter.[4] Though the said interplay can be seen in all the fundamental fields of physical inquiry, it has been quite prominent in 20th century cosmology, partly because of the very nature of its subject-matter, but partly perhaps too because of the kind of factual evidence on which it must rely. I feel therefore that it might be worth while to illustrate with examples from cosmology a few basic questions regarding the interplay between philosophical insights and mathematical theories.

1. The first question that I wish to consider is a fairly obvious one: If we take a look at facts, we see that theories may be said to be *corroborated* or *refuted* by them; what can philosophical insights do for theories? In principle, they can of course determine them, by supplying their axioms. Descartes' cosmological programme (in his *Discourse on Method* , part V) should probably be understood as an attempt in this direction. And in this century, E. A. Milne quite deliberately set out to derive the entire system of the laws of physics from a few insights concerning the structure of the universe. However, Milne's cosmology is afflicted by the following difficulty, which is bound to arise in all enterprises of this kind: in the course of actually building a mathematical theory of nature that is generally adequate to facts but rests altogether on a priori principles, Milne has had to supplement his original allegedly insightful and at any rate philosophically plausible postulates with additional assumptions, that are more or less clearly *ad hoc* and for which he could provide at best a far-fetched justification. From the consideration that "if a rational understanding of the universe is possible, it ought to be possible to set up a consistent system of time-keeping throughout the universe",[5] Milne derives the conception of a family of "fundamental observers", each of which can make with his clock and theodolite a set of observations indistinguishable from those made by any other fundamental observer. Having *proved* that the coordinates employed by three collinear fundamental observers transform into one another by Lorentz transformations, he *postulates* that this holds good also for any set of fundamental observers in three-space.[6] Further assumptions are: that the universe has zero net angular momentum,[7] that the acceleration of a free particle is a unique function of its position, velocity and epoch,[8] etc. The willingness of Milne and the like of him to enlarge the *a priori* basis of their deductions as it might be required for the advancement of their theories has contributed not a little to the current distrust of speculation in natural philosophy.

A somewhat different approach is exemplified by Bondi and Gold's version

of the Steady State cosmology.[9] Here the theory is derived from a single philosophical insight, embodied in the Perfect Cosmological Principle, plus one or two undeniable facts, such as the darkness of the sky at night or the frequency-shift of radiation received from distant sources. The Perfect Cosmological Principle says that the universe is homogeous in space *and* time, so that it looks more or less the same from any spatiotemporal vantage point. It is justified by a typically philosophical argument, reminiscent of Reichenbach's vindication of the straight rule of induction.[10] Unless the looks of nature are always and everywhere approximately the same we would not be entitled to extrapolate to the entire breadth and duration of the universe the results of our exact physical measurements, which are performed within a pitifully small region of it. Thus, *either* the Perfect Cosmological Principle is true, and that disposes of the theories that contradict it, *or* the Perfect Cosmological Principle is false, but then the particular theories that may be developed in opposition to it have scarcely any chance of being true instead. The Perfect Cosmological Principle, together with factual evidence that the larger clusters of matter are receding from one another, implies that most famous or infamous tenet of the Steady State Theory: the Continuous Creation of Matter. It is indeed ironic that a principle introduced to ensure the universal validity of our terrestrial physics should thus lead to the negation of a deeply entrenched and well corroborated local law. Before the discovery of the relic microwave background radiation made it unfashionable, the Steady State Theory was strongly resisted in the name of matter conservation. Thus, Mario Bunge forcefully argued in 1962 that the acceptance of continuous creation would spell disaster for scientifc thought and would make it indistinguishable from magic.[11] Others, I must recall, were charmed by the dialectical piquancy of a theory in which the sheer repetition of *creatio ex nihilo* is the means of securing that *nihil sub sole novum*.[12]

Einstein's General Theory of Relativity bears witness to a more subtle mode of action of insight on theory. Philosophical vision manifestly guides the formulation of the theory, but is not incorporated into it as an axiom or a set of axioms. Moreover, the theory, once formulated, makes the philosophical vision more precise and can, in a sense, even be said to modify it. Answering to E. Kretschmann's remark that general covariance is a trivial requirement that any phyical theory is capable of fulfilling, Einstein declared in 1918 that General Relativity rests on three principles: the Principle of Equivalence, the Principle of General Covariance, and Mach's Principle.[13] If, following Gerald Holton,[14] we classify the diverse ingredients or strands of a piece of scientific

thought into the empirical or factual, the analytic or logico-mathematical, and what Holton calls the thematic element, of which our "philosophical insight" is a proper or an improper part, it is apparent that the Principle of General Covariance is a logico-mathematical requirement, while the Principle of Equivalence is a bold but reasonable extrapolation from facts. The philosophical component of General Relativity must therefore lie mainly with Mach's Principle. Now Mach's Principle is not an axiom of the theory. Indeed, in the particular version proposed by Einstein in 1918, it is not even a theorem of it - though in a modified version it has been said to state the boundary conditions prescribed for the field equations.[15] But the insight which Mach's Principle is supposed to express certainly guided Einstein during the long strenuous search that led to the formulation of General Relativity.[16] In traditional philosophical language that insight can be stated thus: Absolute space and absolute time - as well as absolute spacetime -, without matter, are physically inviable mathematical contraptions. This thesis is implicit in the writings of the greatest pre-Newtonian natural philosophers, Aristotele and Descartes, and was explicity defended by Leibniz against Newton's spokesman, Samuel Clarke.[17] It was also shared by Kant, but this philosopher, believing that the conceptual framework of Newtonian dynamics was a prerequisite of natural science, felt compelled to conclude that nature itself, regarded as a scientific object, was no less unreal, or "transcendentally ideal", than absolute space and time had to be. The said insight also inspired Ernst Mach's celebrated criticism of Newton's scholion on space and time.[18] Einstein traces the origin of his Mach principle to this Machian criticism.[19] Since Newtonian time had been superseded already in the Special Theory of Relativity, the Mach Principle of General Relativity must be directed against Newtonian space. Now the absoluteness of space is manifested, according to Newton, through the phenomena of absolute acceleration.[20] It might seem paradoxical that Newton should resort with such assurance to the concept of an absolute acceleration, while providing no criteria and having no use for the presumably more fundamental concept of absolute motion.

But this paradox was dispelled by Minkowski's interpretation of Special Relativity and the subsequent reformulation of Newtonian theory in four dimensions by Cartan and others.[21] All that is needed to make sense of an absolute acceleration without having to conceive an absolute motion is that spacetime be endowed with a linear connection, plus whatever structure is employed for singling out worldlines, *i.e.* spacetime curves that are nowhere tangent to an hypersurface of simultaneity. The linear connection then determines an intangible yet perfectly rigid network of

geodesic worldlines, which any material particle must follow if let loose at an arbitrary spacetime point. Absolutely accelerated motion is simply motion which deviates from this natural network, *i.e.* motion along a non-geodesic worldline. This reconstruction of the concept of absolute acceleration clearly brings out the effective physical meaning of Newton's talk of absolute space. In Newtonian and in special relativistic dynamics, spacetime, of itself, constrains force-free matter to follow a special kind of cosmic track. On the other hand, the presence of matter makes no difference in the structure of spacetime. According to Einstein, such lopsidedness in the mutual relationship between two physical entities is utterly at variance with everything we know of nature.[22] Such was the insight that Einstein sought to embody in the Mach Principle of 1918. This can be paraphrased as follows: The linear connection, or rather, the spacetime metric on which it depends in a relativistic theory, is fully determined by the distribution of matter. Now, if Special Relativity holds locally, the distribution of matter should be represented by the stress-energy tensor, a symmetric tensor field of order two, whose covariant divergence vanishes. Mach's Principle implies then that the components of the metric relative to a given spacetime chart must be obtained by integration of a system of differential equations relating the components of the stress-energy tensor with those of a tensor field constructed from the metric itself and its derivatives with respect to that chart. If Newton's gravitational theory is valid as a first approximation, the required equations must depend on the second derivatives of the metric. If we assume for simplicity's sake that no higher order derivatives are involved, the stated conditions uniquely determine Einstein's field equations (up to two arbitrary constants).[23] However, unless one understands the "distribution of matter" in a Pickwickian sense, the field equations do not agree with the above version of Mach's Principle, since they also have solutions if the stress-energy tensor is identically zero.[24] Later in life Einstein rejected the 1918 Mach Principle. On February 2, 1954, he wrote to Felix Pirani:

> *One shouldn't talk any longer of Mach's principle, in my opinion. It arose at a time when one thought that 'ponderable bodies' were the only physical reality and that in a theory all elements that are not fully determined by them should be conscientiously avoided. I am quite aware of the fact that for a long time, I, too, was influenced by this fixed idea.*[25]

Nevertheless, he unflinchingly held to the original insight that spacetime could not be allowed to act on matter without the latter acting on spacetime in its turn. On May 12, 1952, he wrote to Max Born, with regard to the apparent failure of one of the three classic effects of

General Relativity:

> *Wenn überhaupt keine Linienablenkung, keine Perihelbewegung und keine Linien-Verschiebung bekannt wäre, wären die Gravitationsgleichungen doch überzeugend, weil sie das Inertialsystem vermeiden (dies Gespenst, das auf alles wirkt, auf das aber die Dinge nicht zurückwirken). Es ist eigentlich merkwürdig, daß die Menschen meist taub sind gegenüber die stärksten Argumenten, während sie stets dazu neigen, Messgenauigkeiten zu überschätzen.*[26]

2. The second question that I wish to consider is, in a way, the inverse of the first. What can theories do for insight? What can mathematical thought contribute to the philosophical understanding of nature? Let us look once more at the interplay between theories and facts. Not only do theories, in connection with their underlying insights, provide the framework for the scientific description of phenomena; they have often predicted the existence of entirely unexpected things, like radio waves or antimatter. Can theories do something analogous for insight? Can pure mathematical thought give birth to a new view of nature? I am not sure I know of a case in which this has happened. Francis M. Cornford once argued that the idea of an absolute space was begotten by Greek geometry.[27] But even if he were right, one might still object that this was not a contribution to insight but to obfuscation. Anyhow, it is evident that mathematics has repeatedly contributed to make existing philosophical views "clear and distinct" and has thereby helped to show in what sense they are tenable. This has occurred in many fields of philosophical inquiry. The importance of mathematics in contemporary logic and foundational research is familiar to everyone. Less well known, but very promising is the use of mathematics in ontology, as in Professor Bunge's pioneering treatise.[28]

In the special domain of our present workshop, few ideas have been more helpful and, I dare say, more fertile, than the mathematical concept of a differentiable manifold, first introduced by Bernhard Riemann for the philosophical purpose of elucidating the essence of physical space,[29] and subsequently perfected by Christoffel, Ricci, Levi-Cività, Weyl, Cartan and others. As an example of what a mathematical theory can do for our understanding of nature, I shall now examine a philosophically significant application of the manifold concept in cosmology. Relativistic cosmology conceives the universe as a four-dimensional real Hausdorff manifold endowed with a Lorentz metric. Such a manifold is called a *spacetime*. The physical properties of the universe are represented by diverse scalar, vector, tensor or spinor fields on spacetime, *i.e.* by smooth mappings of the manifold into several fibre bundles that naturally go with it. (Indeed, the Lorentz metric is one such mapping,

into the bundle of symmetric covariant tensors of order two.) This conceptual framework has paved the way for a totally new approach to one of the oldest and most intractable problems of cosmology, the problem of the spatial extent and the temporal beginning of the universe. When Immanuel Kant, notwithstanding his beautiful early contributions to the development of a Newtonian world-view,[30] pronounced cosmology a pseudoscience, rooted in a fatal illusion of human reason, he counted this problem as the first of the four that forced him to draw this conclusion.[31] Kant maintained that the temporal succession of physical phenomena must have a beginning, because otherwise any current event would mark the end of an eternal process, which he deemed absurd. He also argued, less plausibly, that the universe must be finite in extent, because an infinite totality of simultaneously given things involves a completed yet infinite synthesis, which again, to his mind, was absurd. On the other hand, a universe finite in extent and developing from a definite starting-point raises the specter of an empty space lying beyond its confines and an empty time running before its beginning. Since Kant believed that empty space and time outside the world are no less absurd than a completed infinity or a terminating eternity, he declared cosmology impossible and judged the universe to be a mere idea, which is useful for regulating our scientific inquiries, but which generates inextricable contradictions as soon as it is held to be a genuine physical object, the proper subject-matter of a science of its own. By conceiving the universe as a differentiable manifold relativistic cosmology can deal at once with both horns of the Kantian dilemma. There is no difficulty in conceiving an infinite spacetime on which the matter fields take non zero values right out to infinity. (No difficulty, that is, if one has no qualms about the classical, Cantorean, conception of continua.) However, if one feels prompted by experience to reject the actual infinity of the universe, one can always abide by the other horn, for it is possible to have a spacetime both finite in extent and of bounded duration, without conjuring up an unthinkable void beyond it. We are all familiar with Riemann's conception of a finite yet unlimited world. Einstein's revival of it, in his "Kosmologische Betrachtungen" of 1917, may rightly be said to mark the beginning of modern cosmology.[32] We may regard a spacetime as spatially finite if every point of it lies on some compact spacelike hypersurface which cuts the spacetime into two components. (This characterization must be refined if closed timelike curves are permitted.) Within the theory of differentiable manifolds such a spacetime is not harder to conceive than, say, a cylindrical surface, through each of whose points passes a circle which cuts the

surface in two. On the other hand, the concept of a universe of bounded duration is not without difficulties and deserves more attention. Let us remark first that a spacetime can have many different shapes, provided that each point in it has a neighbourhood that can be charted onto an open subset of R^4. Thus, a spacetime M can fulfil the following condition: for every point P in M and for every future-directed timelike curve c through P, parametrized by proper time, the domain of c is bounded below. If, for simplicity's sake, we ignore the existence of causal links over null curves - these can be dealt with by resorting to a so-called generalized affine parameter, but it would be untimely to go into such refinements now -,[33] it is evident that, if we happened to live in a spacetime such as M, current events would not mark the conclusion of any eternal process. M is therefore immune to Kant's strictures against an eternal past. But M is no preceded by an empty time. In order to see this more clearly, let us say that a point P in M is a *starting-point* of a future-directed timelike curve $c: I \to M$, if for every neighbourhood U of P there is a real number $t_U \in I$, such that $c(t)$ lies in U for every $t \in I$ with $t \leqslant t_U$. We shall say that such a curve is *past-inextendible* if no point in M is a starting-point of it. Now the condition prescribed for M does not mean that every future-directed timelike curve in M has a starting-point, but rather that every *past-inextendible* such curve in M has a domain of definition that is bounded below. That such past-inextendible curves exist in M can be seen by considering a field V of unit timelike vectors on M. (Such a field exists if M, as we may sensibly assume, is time-orientable.) Let c be the maximal integral curve of V through a given point P. The domain of c is bounded below. Let its greatest lower bound be 0. (This can be secured, if necessary, by regraduating the parameter.) If c were defined at 0, $c(0)$ would be its starting-point. But then $c(0)$ would have an open neighbourhood U in M, V would be defined on all U, and the maximal integral curve of V through $c(0)$ would have values in the intersection of U with the past of $c(0)$. Hence c, contrary to our assumption, would not be a maximal integral curve of V through $c(0)$. Therefore c is not defined at 0. Moreover, since c is maximal, it cannot be extended to a curve $\hat{c}$, defined at 0 and agreeing with c on the positive side of 0. Consequently, there is no point in M that is a starting-point of c.

The condition prescribed for our spacetime M obtains in the current favoured Big Bang world-models. Let W be a Big Bang spacetime and let c be any future-directed past-inextendible timelike geodesic in W, parametrized by proper time. Suppose that c is complete, *i.e.* that it

has a value for every real number. Then there must be a real number t such that $c(t)$ is the Big Bang, *i.e.* a point containing all the energy in the universe. At such a point the stress-energy tensor, the curvature, etc. would not be smooth. Since this is impossible by definition, no such point can belong to W. Consequently, c is not complete and t is the greatest lower bound of its domain. The Big Bang is often described as the beginning of the world, or, at least, as the beginning of its present dispensation. We see that this description is misleading. Big Bang universes have their duration bounded below, but do not have a proper beginning. There is in them no *first* instant that might tempt us to ask what went on before it. For every given instant along a causal line there is another instant preceding it, at which something was already going on. However, each event in the past of any given one lies at a finite temporal distance from it. In this sense, in a Big Bang world we can touch, so to speak, with the tip of our fingers -- provided we manage to stretch them back in time some fifteen billion years -- the radical contingency of nature. But an eternal world would be no less contingent, as Leibniz noted in his tract *De rerum originatione radicali*.[34]

At this point, it may be instructive to compare the Big Bang with the singularity found inside a black hole. Take a Schwarzschild field, which is the simplest case in which a black hole can arise, and a Friedmann universe, which is the simplest kind of Big Bang universe. A Schwarzschild field is a symmetric and static solution of Einsteins' equations for empty spacetime. By Birkhoff's theorem, the solution holds even if the requirement of staticity is relaxed, provided perfect symmetry is preserved at all times. The Schwarzschild solution involves a constant of integration m, which is usually and very naturally interpreted as the mass of the material source of the field.[35] If all the matter in the source lies within a sphere of radius less than $2Gm/c^2$ -- where G is the gravitational constant and c is the speed of light *in vacuo* -- it must collapse to a space point at the axis of symmetry of the field: the black hole singularity. At such a point the matter fields, the curvature, the metric, etc. cannot be smooth, so we are led to conclude, once more, that there is no such point in our manifold. A Schwarzschild field must therefore be regarded as a spacetime punctured, or rather scratched, at its axis of symmetry. However, most physicists will not allow the mass m simply to vanish into the singularity. Of course, one can strictly conceive of the Schwarzschild field about the singularity as the field left behind by a collapsing, *i.e.* literally self-annihilating mass. The field is fully determined by the boundary conditions.

The idea that it must break down as soon as its source is no longer there to support it bespeaks an inability to think spatiotemporally and field-theoretically. But there is something philosophically disquieting about a chunk of matter thus fading away into nothingness. Hence the common feeling that black holes constitute a genuine paradox in General Relativity and can only be understood, if at all, from a post-relativistic, probably quantumtheoretical perspective. On the other hand, a Friedmann universe is derived from some simplifying assumptions concerning the current global distribution of matter. In such a universe, as one goes backward in time -- say as one moves towards the past along any parametric line of the Robertson-Walker time coordinate -- the density of matter indefinitely increases in one's neighbourhood. Indeed, if ϱ denotes the density component of the stress-energy tensor (in terms, say, of the Robertson-Walker coordinates) and c is a parametric line of the time coordinate, parametrized by Robertson-Walker time, the composite function $\varrho \cdot c$ grows beyond all bounds as the argument approaches a definite real number t_0. Consequently ϱ would be infinite at the spacetime point $c(t_0)$ if c were defined at t_0, that is to say, if t_0 were in the range of the Robertson-Walker time coordinate. But ϱ cannot become infinite at any point of spacetime, and hence we must conclude that $c(t_0)$ does not exist and that t_0 is not in the range of the time coordinate. However, this should not be objectionable even to those who would reject matter annihilation inside black holes, for the source of the Friedmann field is coeval with the field itself, so that the excision of the Big Bang singularity is not paradoxical in the way in which the excision of black hole singularities might be said to be. This is not to deny that a post-relativistic theory explaining black holes will probably throw a light on the Big Bang too and might eventually do away with it. Be that as it may, the preceding has made clear, I hope, that by means of Riemann's idea of a manifold one can conceive without paradox an unbounded yet finite universe, which has an age though it never was born.

3. I can barely touch on a third question that must be asked with regard to insights and theories. To clarify it I resort once again to the analogy of the interplay between theories and facts. There is no doubt that, even if there are no incorrigible observation statements and all phenomena can be described in different ways within diverse conceptual frameworks, there is something final about facts, by virtue of which they provide a touchstone for theories. Thus, now that all those stones have been brought back from the moon, one would not wish to maintain with Aristotle that it is made of incorruptible, pellucid, imponderable ether.

Is there a comparable finality about insights? Is there any way of arriving at definitive philosophical insights that set fixed boundaries to admissible theories? Some of the greatest philosophers have thought they had found a method for this and have devoted considerable efforts to its pursuit. In the last two centuries the most influential of them has doubtless been Kant, who made the following simple yet alluring discovery: if one can gain an insight into the prerequisites of scientific knowledge one will thereby obtain an insight into its subject-matter, for, as he put it, "die Bedingungen der Möglichkeit der Erfahrung überhaupt sind zugleich Bedingungen der Möglichkeit der Gegenstände der Erfahrung."[36] He believed he could claim, on these grounds, that Euclidean geometry and Newtonian chronometry, the continuity of qualitative change, mass conservation, causal determinism and the equality of action and reaction between distant bodies, were permanent features of physical science. As is well known, not one of them survived the advent of Relativity and the Quantum. But Kant's failure in the application of his method does not invalidate the method itself, and arguments in Kantian style are still used for vindicating basic scientific assumptions. I have already mentioned Bondi and Gold's justification of the Perfect Cosmological Principle. The following example is taken from Hawking and Ellis' treatise, *The Large Scale Structure of Space-Time*. Discussing the possibility that a relativistic spacetime might contain closed timelike curves - *i.e.* timelike curves homeomorphic to a circle - the authors write:

> *However the existence of such curves would seem to lead to the possibility of logical paradoxes: for, one could imagine that with a suitable rocketship one could travel round such a curve and, arriving back before one's departure, one could prevent oneself from setting out in the first place. Of course there is a contradiction only if one assumes a simple notion of free will; but this is not something which can be dropped lightly since the whole of our philosophy of science is based on the assumption that one is free to perform any experiment.*[37]

Hawking and Ellis allude to H. Schmidt's "Model of an oscillating cosmos that rejuvenates during contraction",[38] which contains closed timelike curves and in which the concept of free will is modified, but conclude that

> *One would be much more ready to believe that space-time satisfies what we shall call the chronology condition: namely, that there are no closed timelike curves.*[39]

The chronology condition is one of the hypotheses of an important theorem on the existence of singularities in spacetime, discovered by Hawking and Penrose in 1970.[40] Now, though it is certainly encouraging to hear two distinguished scientists argue, in a strictly scientific argument, from

the premise that freedom is a prerequisite of science, one must also bear in mind that, as a matter of fact, one is not free to perform any experiment anywhere at any time. Indeed, physics must constantly extrapolate from our terrestrial laboratories to spacetime regions where free experimentation is physically impossible. Consequently, Hawking and Ellis' argument must imply either that science is illusory or that its possibility requires only that *some* timelike curves be open, and that they include the worldlines of the materials out of which scientists and scientific instruments are made. (By the way, the compactness of *some* timelike curves is not more inimical to freedom than the existence of a Cauchy surface in spacetime, which Hawking and Ellis do not seem to be willing to exclude *a priori*, though they readily admit that "there does not seem to be any physically compelling reason for believing that the universe admits" one.)[41] The Kantian method of philosophical inquiry into the prerequisites of science can also be said to inspire the current German school of protophysics, but a discussion of their endeavours would be somewhat out of place here, for not only do they show little sympathy with all that Relativity philosophically stands for, but they tend to dismiss cosmological theories as metaphysical fancies.[42]

A completely different approach to the subject of philosophical insight was initiated by Edmund Husserl about 1900, giving rise to the so-called Phenomenological Movement. Husserl maintained that we can grasp the features that make up the "essence" *(Wesen)* of phenomena, quite independently of whether the entities seemingly manifested by those phenomena actually exist or not. Such "grasp of essences" *(Wesenschau)* can be attained by freely varying in imagination the phenomena actually experienced, the "essential" traits being precisely those that remain invariant under free variation. Husserl's approach exerted a considerable influence on the German and Central European *Geisteswissenschaften* after the First World War and is now in vogue in American psychology, but its effect on the philosophy of physics has been negligible. Nevertheless, it is undeniable that a flair for such constant features of phenomena, for what Husserl called "den invarianten allgemeinen Stil, in dem [unsere empirisch anschauliche Welt] im Strömen der totalen Erfahrung verharrt",[43] has always been a critically important ingredient of scientific genius. Scientists who, in Max Born's words, "haben gar kein Gefühl für die innere Wahrscheinlichkeit einer Theorie",[44] have seldom made decisive contributions to natural philosophy. And evidently such a feeling for the intrinsic likelihood of theories cannot grow only out of a thorough acquaintance with available mathematical devices and experimental results. Let me give one example of what I take to be Husserl's meaning.

It is usually assumed that the spacetime manifold of a physically viable relativistic world-model must be time-orientable. Mathematically spoken: the manifold must admit an everywhere defined smooth field of timelike vectors. I should say that this assumption does not rest on the observation of particular facts, which could never authorize such a sweeping generalization, but on an intuitive grasp of the apparent necessity that every event *E* be surrounded by a neighbourhood inside which the events to the future of *E* can be neatly discerned from the events to the past of *E* (I note in passing that the time-orientability of spacetime must not be confused with the so-called time asymmetry of some physical phenomena; the latter, as illustrated, *i.e.*, by heat flow, cannot even be described unless time-orientation is taken for granted, for it consists in the fact that certain types of events, A, B, C,... always happen so that C is to the future of B when A is to its past, and never take place in the inverse order.) However, we ought to be wary of relying too freely on our grasp of essences. I dare say that all of us would readily assume that each point-event in the universe has a sizeable neighbourhood that is contractible to it, if we were not acquainted with physical speculations involving multiply connected spacetimes.[45] Even if these speculations do not look promising, they are not absurd, and we may not simply discard them as physically impossible just because they defy our sense of plausibility. On such matters, the early reception and eventual acceptance of Einstein's Relativity have taught us a lesson that is not easily forgotten: a theory can contradict some of our most cherished beliefs concerning the nature of things and yet turn out to agree with our genuine intuitions much better than the theory it supersedes.

Summing up: neither Kant's nor Husserl's method can provide foolproof philosophical insights, though of course, if one is sure that he has lighted on a true prerequisity of scientific knowledge, or that he has effectively grasped an invariant feature of the world, he will certainly hold his discovery binding for science. The safest test of insights appears to be, not their self-evidence, which can be deceptive, but the possibility of articulating them in a mathematical theory that yields a satisfactory framework for the understanding of facts. In the end, as it often happens philosophy, our discussion has not revealed us anything that we did not know beforehand, but has only clarified what we had always known: while facts depend on insightful theories for their coherent formulation, insights depend on theoretically conceived facts for the corroboration of their adequacy. Mathematical theorizing, at their prompting, weaves the web that holds insights and facts together.

Notes

1. M. Born, *Experiment and Theory in Physics*, Cambridge: at the University Press, 1943.

2. A. Einstein, H. und M. Born, *Briefwechsel 1916-1955*, München: Nymphenburger Verlagshandlung, 1969, p. 203. I thank the Estate of Albert Einstein for allowing me to reproduce the above and the remaining passages of Einstein's writings quoted in this paper.

3. Einstein-Born Briefwechsel, l.c., p. 204.

4. See, for example, N. Maxwell, "The rationality of scientific discovery", *Phi. Sci.*, 41 (1974) 123, 247; J. Agassi, *Science in Flux*, Dordrecht: Reidel, 1975; L.Laudan, *Progress and its Problems*, Berkeley: University of California Press, 1977.

5. E. A. Milne, *Modern Cosmology and the Christian Idea of God*, Oxford: Clarendon Press, 1952, p. 50.

6. E. A. Milne, *Kinematic Relativity*, Oxford: Clarendon Press, 1948, p. 39.

7. E. A. Milne, *Modern Cosmology and the Christian Idea of God*, p. 61, compare, however, *Kinematic Relativity*, p. 93.

8. E. A. Milne, *Kinematic Relativity*, p. 64.

9. H. Bondi and T. Gold "The Steady-State Theory of the Expanding Universe", *Mon. Not. R. Astron. Soc.*, Cambridge: at the University Press, 1960, Chapter XII.

10. H. Reichenbach, *The Theory of Probability*, Berkeley: University of California Press, 1949, paragraph 91.

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12. Compare the sympathetic stance towards the Steady State Theory taken in J. Merleau-Ponty's *Cosmologie du XXe Siecle*, Paris: Gallimard, 1965.

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15. J. A. Wheeler, "Mach's Principle as Boundary Condition for Einstein Equations", in Chiu and Hoffmann, eds, *Gravitation and Relativity*, New York: W. A. Benjamin, 1964, pp. 303 - 349.

16. Compare A. Einstein, "Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen", *Jahrb. Radioakt. u. Elektronik*, 4 (1907), p. 454; "Zum gegenwärtigen Stande des Gravitationsproblems", *Phys. Ztschr.* 14 (1913), pp. 1254 f., 1260 f., 1265; A. Einstein and M. Grossmann, *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation*, Leipzig: Teubner, 1913, p. 6. See also Mach's obituary, by Einstein in *Phys. Ztschr.*, 17 (1916) 101.

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